

SUMMER PACKET for Algebra students entering Geometry

Please complete every problem and SHOW ALL WORK. NO WORK = NO CREDIT. Write your answers on the answer sheet at the end of the packet. This assignment will be graded for both accuracy and completion. You will be tested during the second week of school on each of the following concepts.

DETERMINING WHETHER A POINT IS ON A LINE

Example 1

Decide whether (3,-2) is a solution of the equation $y = 2x - 8$

$$-2 = 2(3) - 8 \quad \text{Substitute 3 for } x \text{ and } -2 \text{ for } y.$$

$$-2 = -2 \quad \text{Simplify.}$$

The statement is true, so (3,-2) is a solution of the equation $y = 2x - 8$

Exercises: Decide whether the given ordered pair is a solution of the equation.

1. $y = 6x + 4; (-2, 8)$ _____

4. $y = \frac{3}{2}x + 10; (4, 12)$ _____

2. $y = -10x - 2; (1, -12)$ _____

5. $y = \frac{5}{9}x + 34; (-9, 27)$ _____

3. $y = -\frac{1}{4}x - 18; (-4, -17)$ _____

6. $y = \frac{2}{3}x - 6; (9, 0)$ _____

CALCULATING SLOPE

Example 2

Find the slope of a line passing through (3,-9) and (2,-1).

$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{Formula for slope}$$

$$m = \frac{-1 - (-9)}{2 - 3} = \frac{-1 + 9}{-1} \quad \text{Substitute values and simplify.}$$

$$m = \frac{8}{-1} = -8 \quad \text{Slope is } -8.$$

Exercises: Find the slope of the line that contains the points

7. (4,1), (3, 6) _____

9. (5, 6), (9,8) _____

11. (-1, 7), (-3, 18) _____

8. (-8, 0), (5, -2) _____

10. (0,-4), (7,3) _____

12. (-6, -4), (1, 10) _____

FINDING THE EQUATION OF A LINE**Example 3**

Find an equation of the line that passes through the point (3, 4) and has a y-intercept of 5.

$$y = mx + b \quad \text{Write the slope-intercept form.}$$

$$4 = 3m + 5 \quad \text{Substitute 5 for } b, 3 \text{ for } x, \text{ and 4 for } y.$$

$$-1 = 3m \quad \text{Subtract 5 from each side.}$$

$$\frac{-1}{3} = m \quad \text{Divide each side by 3.}$$

The slope is $m = \frac{-1}{3}$. The equation of the line is $y = \frac{-1}{3}x + 5$

Exercises: Write the equation of the line that passes through the given point and has the given y-intercept.

13. (2, 1); $b=5$ _____

16. (7, 0); $b=13$ _____

14. (-5, 3); $b=-12$ _____

17. (-3, -3); $b=-2$ _____

15. (-3, 10); $b=8$ _____

18. (-1, 4); $b=-8$ _____

FINDING THE EQUATION OF A LINE**Example 4**

Write an equation of the line that passes through the points (4, 8) and (3, 1). Find the slope of the line.

$$m = \frac{1-8}{3-4} \quad \text{Substitute values.}$$

$$m = \frac{-7}{-1} = 7 \quad \text{Simplify.}$$

$$1 = 7(3) + b \quad \text{Substitute values into } y = mx + b.$$

$$1 = 21 + b \quad \text{Multiply.}$$

$$-20 = b \quad \text{Solve for } b.$$

Exercises: Write an equation of the line that passes through the given points.

19. (6, -3), (1, 2) _____

21. (5, -1), (4, -5) _____

23. (-3, -7), (0, 8) _____

20. (-7, 9), (-5, 3) _____

22. (-2, 4), (3, -6) _____

24. (1, 2), (-1, -4) _____

Name _____

DISTANCE FORMULA**Example 5**

Find the distance between
the points $(-4, 3)$ and $(-7, 8)$

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-7 - (-4))^2 + (8 - 3)^2} \\ &= \sqrt{(-3)^2 + (5)^2} \\ &= \sqrt{34} \end{aligned}$$

Exercises: Find the distance between the points

25. $(3, 6), (0, -2)$ _____

27. $(-3, 4), (1, 4)$ _____

29. $(8, -2), (-3, -6)$ _____

26. $(5, -2), (-6, 5)$ _____

28. $(-6, -6), (-3, -2)$ _____

30. $(-8, 5), (-1, 1)$ _____

COMBINING LIKE TERMS**Example 6**

Simplify

$8x^2 + 16xy - 3x^2 + 3xy - 3x$

$8x^2 - 3x^2 + 16xy + 3xy - 3x$

$5x^2 - 3x + 19xy$

*Group like terms**Simplify***Exercises: Simplify.**

31. $6x + 11y - 4x + y$

33. $-3p - 4t - 5t - 2p$

35. $3x^2y - 5xy^2 + 6x^2y$

32. $-5m + 3q + 4m - q$

34. $9x - 22y + 18x - 3y$

36. $5x^2 + 2xy - 7x^2 + xy$

SOLVING EQUATIONS WITH VARIABLES ON BOTH SIDES**Example 7**

Solve.

$$6a - 12 = 5a + 9$$

$$a - 12 = 9$$

Subtract 5a from both sides

$$a = 21$$

*Add 12 to each side***Exercises: Solve the equation.**

37. $3x + 5 = 2x + 11$

38. $8m + 1 = 7m - 9$

39. $11q - 6 = 3q + 8q$

40. $-14 + 3a = 10 - a$

41. $-2t + 10 = -t$

42. $-7x + 7 = 2x - 11$

SOLVING INEQUALITIES**Example 8**

Solve.

a. $5x - 4 \geq 4x + 6$

b. $10 - 7x < 24$

When you multiply or divide each side of an inequality by a *negative* number, you must *reverse* the inequality symbol to maintain a true statement.

a. $5x - 4 \geq 4x + 6$

$$x - 4 \geq 6$$

$$x \geq 10$$

b. $10 - 7x < 24$

$$-7x < 14$$

$$x > -2$$

Exercises: Solve the inequality.

43. $-x + 2 > 7$

44. $-5 + m < 21$

45. $z + 6 > -2$

46. $c - 18 < 10$

47. $x - 5 < 4$

48. $-3x + 4 \leq -5$

WRITING AND SIMPLIFYING RATIOS**Example 9**

- a. Train A takes 35 minutes to travel its route. Train B, traveling the same route but making more stops, takes 47 minutes. What is the ratio of the time of Train A to Train B?
- b. Jennie's height is 4 feet, 7 inches. Her younger sister's height is 25 inches. Find the ratio of Jennie's height to her sister's.

Solutions

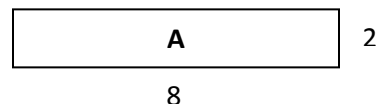
a. 35 minutes to 47 minutes = $\frac{35 \text{ minutes}}{47 \text{ minutes}} = \frac{35}{47}$

b. Convert 4 feet, 7 inches to inches: $4(12) + 7 = 55$ inches

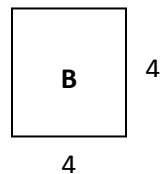
$$55 \text{ inches to } 25 \text{ inches} = \frac{55 \text{ inches}}{25 \text{ inches}} = \frac{55}{25} = \frac{11}{5}$$

Exercises: Write the following ratios.

49. Basmati rice needs to cook for 20 minutes, while quinoa (another grain) cooks for 25 minutes. What is the ratio of cooking times for rice to quinoa?
50. Jonathan caught 7 fish and Geogeanne caught 4. What is the ratio of fish caught of Jonathan to Geogeanne?
51. Two sunflowers' growth was measured daily. At the end of the experiment, Sunflower A had grown from 2 inches to 2 feet, 3 inches. Sunflower B had grown from 3 inches to 2 feet, 6 inches. Find the ratio of the growth in height of Sunflower A to Sunflower B.

Use the diagram at the right.

52. What is the ratio of length to width of rectangle A?
53. What is the ratio of the perimeter of rectangle A to the perimeter of rectangle B?
54. What is the ratio of the area of rectangle A to the area of rectangle B?

**DISTRIBUTIVE PROPERTY****Example 10**

Solve.

a. $4(x + 3) = 36$
 $4x + 12 = 36$
 $4x = 24$
 $x = 6$

b. $6(x + 4) + 12 = 5(x + 3) + 7$
 $6x + 24 + 12 = 5x + 15 + 7$
 $6x + 36 = 5x + 22$
 $x = -14$

Exercises: Solve.

55. $2(x + 7) = 20$

56. $-10(y + 8) - 40$

57. $7(2 - x) = 5x$

58. $-4(x - 6) = 28$

SOLVING PROPORTIONS**Example 11**

Solve.

$$\begin{aligned} \text{a. } \frac{x}{8} &= \frac{3}{4} \\ 4x &= 8 \cdot 3 \\ 4x &= 24 \\ x &= 6 \end{aligned}$$

$$\begin{aligned} \text{b. } \frac{6}{x+4} &= \frac{1}{9} \\ 6 \cdot 9 &= x + 4 \\ 54 &= x + 4 \\ 50 &= x \end{aligned}$$

Exercises: Solve.

59. $\frac{y}{50} = \frac{3}{100}$

60. $\frac{6}{45} = \frac{2z + 10}{15}$

61. $\frac{3}{p - 6} = \frac{1}{p}$

62. $\frac{3}{8} = \frac{3}{2d}$

63. $\frac{1}{18} = \frac{5}{-4(x - 1)}$

64. $\frac{r}{3r + 1} = \frac{2}{3}$

65. $\frac{3w + 6}{28} = \frac{3}{4}$

66. $\frac{3}{m + 4} = \frac{9}{14}$

67. $\frac{w}{4} = \frac{9}{w}$

SIMPLIFYING RADICALS**Example 12**Simplify the expression $\sqrt{20}$

$$\begin{aligned} \sqrt{20} &= \sqrt{4} \cdot \sqrt{5} \\ &= 2\sqrt{5} \end{aligned}$$

Exercises: Simplify the expression.

68. $\sqrt{121}$

69. $\sqrt{40}$

70. $\sqrt{243}$

71. $\sqrt{52}$

72. $\sqrt{27}$

73. $\sqrt{288}$

74. $\sqrt{45}$

75. $\sqrt{80}$

76. $\sqrt{320}$

77. $\sqrt{72}$

78. $\sqrt{50}$

79. $\sqrt{225}$

SIMPLIFYING RADICAL EXPRESSIONS**Example 13**

$$\begin{aligned} \text{a. } 5\sqrt{3} - \sqrt{3} - \sqrt{2} \\ = 4\sqrt{3} - \sqrt{2} \end{aligned}$$

$$\begin{aligned} \text{b. } (2\sqrt{2})(5\sqrt{3}) \\ = 2 \cdot 5 \cdot \sqrt{2} \cdot \sqrt{3} \\ = 10\sqrt{6} \end{aligned}$$

$$\begin{aligned} \text{c. } (5\sqrt{7})^2 \\ = 5^2 \sqrt{7^2} \\ = 25 \cdot 7 \\ = 175 \end{aligned}$$

Exercises: Simplify the radical expression.

80. $\sqrt{75} + \sqrt{3}$

81. $-\sqrt{147} - \sqrt{243}$

82. $(5\sqrt{4})(2\sqrt{4})$

83. $\sqrt{50} - \sqrt{18}$

84. $(3\sqrt{14})(\sqrt{35})$

85. $(6\sqrt{5})^2$

86. $\sqrt{64} - \sqrt{28}$

87. $(\sqrt{363})(\sqrt{300})$

88. $(4\sqrt{2})^2$

89. $\sqrt{44} + 2\sqrt{11}$

90. $(\sqrt{32})(\sqrt{2})$

91. $(8\sqrt{3})^2$

92. $\sqrt{125} - \sqrt{80}$

93. $(\sqrt{98})(\sqrt{128})$

94. $(10\sqrt{11})^2$

95. $\sqrt{242} + \sqrt{200}$

SIMPLIFYING QUOTIENTS WITH RADICALS**Example 14**Simplify the quotient $\frac{6}{\sqrt{5}}$

$$\begin{aligned} \frac{6}{\sqrt{5}} &= \frac{6}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} \\ &= \frac{6\sqrt{5}}{\sqrt{5}\sqrt{5}} \\ &= \frac{6\sqrt{5}}{5} \end{aligned}$$

Exercises: Simplify the quotient.

96. $\frac{4}{\sqrt{3}}$

97. $\frac{2\sqrt{3}}{\sqrt{5}}$

98. $\frac{\sqrt{32}}{\sqrt{5}}$

99. $\frac{5}{\sqrt{7}}$

100. $\frac{\sqrt{12}}{\sqrt{24}}$

101. $\frac{\sqrt{27}}{\sqrt{45}}$

102. $\frac{2\sqrt{3}}{\sqrt{6}}$

103. $\frac{\sqrt{18}}{\sqrt{10}}$

104. $\frac{\sqrt{50}}{\sqrt{75}}$

SOLVING LITERAL EQUATIONS**Example 16**

Given the formula for the surface area of a right cylinder, solve for h . $S = 2\pi r^2 + 2\pi rh$

$$S = 2\pi r(r + h)$$

$$\frac{S}{2\pi r} = r + h$$

$$\frac{S}{2\pi r} - r = h$$

or

$$S - 2\pi r^2 = 2\pi rh$$

$$\frac{(S - 2\pi r^2)}{2\pi r} = h$$

Exercises: Solve the literal equation for the indicated variable. Assume variables are positive.

105. $V = \frac{4}{3}\pi r^3; r$

106. $V = s^3; s$

107. $V = \pi r^2 h; h$

108. $A = \frac{1}{2}bh; h$

109. $P = 2l + 2w; l$

110. $S = 6s^2; s$

111. $A = \frac{1}{2}h(b_1 + b_2); b_1$

112. $V = lwh; h$

113. $a^2 + b^2 = c^2; b$

ALGEBRAIC EXPRESSIONS**Example 17**

a. Write an expression for seven less than a number

$$x - 7$$

b. Write an equation for three times less than six times a number is five times the same number plus 5, then solve.

$$6x - 3 = 5x + 5$$

$$x - 3 = 5$$

$$x = 8$$

Exercises: Write the expression or equation. Solve the equations.

114. Half of a number plus three times the number

115. The product of five and a number decreased by seven equals thirteen.

116. Sixteen less than twice a number is 10.

117. Twice a number increased by the product of the number and fourteen results in forty-eight.

118. Half of a number is three times the sum of the number and five.

PERCENT PROBLEMS**Example 18**

- a. What number is 12% of 75?

$$x = 0.12(75)$$

$$x = 9$$

- b. 6 is what percent of 40?

$$6 = 40p$$

$$0.15 = p$$

$$p = 15\%$$

Exercises:

119. What number is 30% of 120?

120. 11 dogs is what percent of 50 dogs?

121. What distance is 15% of 340 miles?

122. 200 is what percent of 50?

123. 34 is what percent of 136?

124. 8 weeks is what percent of a year?

SIMPLIFYING RATIONAL EXPRESSIONS**Example 19**

Simplify.

a.
$$\frac{8x^2+12x}{4x^2+16x} = \frac{4x(2x+3)}{4x(x+4)} = \frac{2x+3}{x+4}$$

b.
$$\frac{y^2-9}{y^2+6y+9} = \frac{(y+3)(y-3)}{(y+3)(y+3)} = \frac{y-3}{y+3}$$

Exercises: Simplify.

125.
$$\frac{5x}{10x^2}$$

126.
$$\frac{14d^2 - 2d}{6d^2 + 8d}$$

127.
$$\frac{-5h + 1}{h + 1}$$

128.
$$\frac{16a^3}{8a}$$

129.
$$\frac{2y - 12}{24 - 2y}$$

130.
$$\frac{t^2 - 1}{t^2 + 2t + 1}$$

131.
$$\frac{(5x^2 + x)}{(5x + 1)}$$

132.
$$\frac{36s^2 - 4s}{4s^2 - 12s}$$

133.
$$\frac{m^2 - 4m + 4}{m^2 - 4}$$

ANSWER SHEET

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|-----------|-----------|-----------|-----------|
| 1. _____ | 25. _____ | 49. _____ | 73. _____ |
| 2. _____ | 26. _____ | 50. _____ | 74. _____ |
| 3. _____ | 27. _____ | 51. _____ | 75. _____ |
| 4. _____ | 28. _____ | 52. _____ | 76. _____ |
| 5. _____ | 29. _____ | 53. _____ | 77. _____ |
| 6. _____ | 30. _____ | 54. _____ | 78. _____ |
| 7. _____ | 31. _____ | 55. _____ | 79. _____ |
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| 10. _____ | 34. _____ | 58. _____ | 82. _____ |
| 11. _____ | 35. _____ | 59. _____ | 83. _____ |
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